The Direct Breaking of Internal Waves at Steep Topography

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Abstract

Internal waves are often observed to break close to the bathymetry at which they are generated, or from which they scatter. This breaking is often very spectacular, with turbulent structures observed hundreds of meters above the seafloor, and driving turbulence dissipations and mixing up to 10,000 times open-ocean levels. An overview is given of efforts to observe and understand this turbulence, and to parameterize it near steep “supercritical” topography (i.e. topography that is steeper than internal wave energy characteristics). Using numerical models, we demonstrate that an important turbulence-producing phenomenon is arrested lee waves. Analogous to a hydraulic jump in water flowing over an obstacle in a stream, these are formed each tidal cycle and break. Similar lee waves are also seen in the atmosphere and shallow fjords, but in those cases their wavelengths are of similar scale to the topography, whereas here they are quite small compared to the water depth and topography. The simulations indicate that these lee waves propagate against the generating flow (usually the tide) and are arrested because they have the same phase speed as the oncoming flow. This allows us to estimate their size a-priori, and, using a linear model of internal tide generation, estimate how much energy they trap and turn into turbulence. This approach has proven to yield an accurate parameterization of the mixing in numerical models, and these models are being used to guide a new generation of observations.
**Introduction**

Internal waves are a mechanism by which energy is transmitted to the interior of the ocean. In order to affect the mean flow and the mixing of nutrients in the ocean, these waves must, by some mechanism, break and become turbulent. The resulting mixing creates lateral density gradients that drive ocean currents, both local, and on the scale of the global thermohaline circulation. It has been estimated that the deep ocean requires 2 TW of energy to maintain the overturning circulation observed, and that this energy can be supplied approximately equally by internal waves forced by the wind and tides (Munk & Wunsch, 1998). Tracking the energy pathways of internal waves generated by winds is difficult because they are spatially variable and sporadic (Alford, 2003; D'Asaro, 1995), and accordingly recent attention has focused on understanding the energy balance of tides.

![Figure 1](image.png)

**Figure 1:** A numerical simulation of internal tide generation from a steep ridge (at x=0 km), and impacting a remote continental margin (at x=1300 km). Velocity is colored. The upper panel is early in the simulation to show the propagation of low-mode internal tides across the basin. The low-modes travel faster than the high-modes, which make up the “ray”-like character of the velocity closer to the ridge. Eventually these rays would fill the whole basin if the resolution of the model permitted. The full solution shows the interaction of the propagating and reflected waves, and indicates the turbulence that would be observed at the remote continental slope.

Though simpler than wind-driven waves, the energy balance of internal tides is still quite complex (Figure 1). Internal tides are generated when the surface tide pushes density-stratified water over topography, creating internal pressure gradients that drive...
the waves. In the bounded ocean, the waves propagate away from the topography in
vertical “modes.” The lowest mode has longest vertical wavelength, the strongest
velocities at the surface and seafloor, and a null in the middle of the water-column, and
propagates with a horizontal phase speed that is faster than higher modes. This can be
seen in Figure 1a, early in a numerical tidal simulation, where the low-modes have
traveled almost 500 km from the ridge at $x=0$, but the more complicated “high-mode”
structure is just starting to form near the ridge. The high modes may break into
turbulence, either locally where they are generated, at remote topography (Figure 1b,
Nash et al., 2004), or by wave-wave interactions in the interior (Henyey et al., 1986;
Polzin, 2009; MacKinnon & Winters, 2005). Low modes can reflect, interfere with one
another, and scatter into higher modes, though their fate is not yet well understood.

An important factor in the study of topographic interactions with internal waves is
the steepness of the topography (Garrett & Kunze, 2007) As seen emanating from the
topography in Figure 1, internal waves organize their energy along “beams” of energy
that propagate at characteristic angles that depend on the frequency of the wave and the
density stratification. If the topography is gentler than these characteristic slopes, it is
said to be “subcritical;” if steeper, it is said to be “supercritical”. Of course real
topography has regions of both, but the extremes are useful because mathematical
predictions of internal tide generation can be formulated for either assumption (for
subcritical topography see (Balmforth et al., 2002; Bell, 1975); for supercritical
topography see (LlewellynSmith & Young, 2003; St. Laurent et al., 2003).

At regions of subcritical topography, such as the Brazil Basin, the generation of
internal tides is relatively linear. The nonlinear interaction of the waves cascades energy
to higher wavenumbers until they break, leading to enhanced dissipation above the sea-
floor (Polzin et al., 1997; St. Laurent et al., 2001). This mechanism has been numerically
simulated recently (Nikurashin & Legg, 2011) and has been parameterized by Polzin,
(2009) based on the topography, the surface tide forcing, and the density stratification.
Using these approaches, a modest fraction of the locally generated energy dissipates near
such small amplitude but rough topography (about 30%, St. Laurent & Nash, 2003), and
the rest is believed to radiate away as low-modes. These approaches agree with available
This paper reviews our efforts to understand the other extreme of “supercritical” topography, when the topography is relatively steep compared to the internal tide rays. In this case, energy is believed to “break” directly near the topography due to non-linearities in the generation process itself. This short-circuits the energy cascade we associate with waves emanating from subcritical topography, and leads to spectacular localized turbulence. This paper reviews observations of these local breaking processes, and documents initial efforts to model and develop simple parameterizations of them. The observed processes are very non-linear, and thus very sensitive to small changes in local forcing, so a large number of caveats come into play before we fully understand dissipation at supercritical sites.

Figure 2: Observations of breaking waves made at Hawaii, on the ridge crest between Oahu and Kauai (modified from Klymak et al 2008). Contours are density surfaces versus time during an onslope phase of the tide. Colors are turbulence dissipation rate, a measure of turbulent strength, as inferred from the size of the breaking overturns in the waves, which almost reach 250 m tall near 06:00.

**Observations**

Breaking of internal waves at abrupt topography has now been observed at a number of locations, but the mechanism leading to this breaking has remained elusive. During the Hawaiian Ocean Mixing Experiment, at the Hawaii Ridge, it was noted that there was strong turbulence near the seafloor extending hundreds of meters into the water column.
(Figure 2; Aucan et al., 2006; Klymak et al., 2008; Levine & Boyd, 2006). These motions drove breaking internal waves that were up to 200 m tall, and dissipation rates up to four orders of magnitude greater than the open-ocean values at those depths.

Considering an event in detail (Figure 2), the turbulence tends to be at the sharp leading face of rising density surfaces (isopycnals). This leading edge breaks, in this example in two patches (centered at 0500 and 0630). During the relaxation of the tide, there is a rebound at mid depth (0700), and sharp oscillations deeper (though these only have modest turbulence). In all, the strong turbulence lasts over three hours; the remaining nine hours of the tidal cycle are relatively quiescent.

The main findings from the observations cited above at Hawaii were:

1. Breaking waves were phase-locked to the surface tide forcing.
2. Turbulence dissipation rates (a measure of the strength of the turbulence) scaled cubically with the spring-neap modulation of the surface-tide forcing.
3. The turbulence that had these characteristics was confined to within a few hundred meters of the seafloor; turbulence further aloft only varied weakly with the tidal forcing.

Similar observations have been made at the Oregon slope (Figure 3, Nash et al., 2007) and the South China Sea Continental Slope (Klymak et al., 2011). The dissipations at these locales were less than at Hawaii, but again depend on the tidal forcing. The phenomenology along these slopes is complicated by both rougher topography, and the presence of shoaling internal tides that originated from elsewhere. At the Oregon slope, the strongest turbulence is during the transition from onslope to offslope flow, in contrast to the ridge crest at Hawaii where the strongest turbulence is during the onslope flow.

The difference in the timing of the turbulence likely depends on the local curvature of the topography and the mooring location relative to local generation sites, though a full explanation of the phenomenology on complex topography has not yet been accomplished.
Figure 3: Breaking waves on the Oregon continental slope. Driven by a combination of barotropic and baroclinic tides that flow over small-scale bumps (of order 250-1000 m horizontal scale) that overlay the otherwise steep continental slope, these waves occur on the leading edge of the down-slope phase of the tide. (modified from Nash et. al. 2007). Upper panel is turbulence dissipation rate with isopycnals overlain; middle panel is isopycnal strain, measuring how stretched or compressed parts of the water column are by the internal tide; bottom pane is the upslope semi-diurnal tidal velocities. The velocity shows downward phase propagation with time, indicating upwards propagation of energy from the sea floor.

A recent set of observations, were undertaken at Luzon Strait between Taiwan and Luzon Island (Figure 4). Luzon Strait is unique in that it consists of two ridges, a short one to the west, and a tall one to the east. It also is somewhat unique because the daily (diurnal) tide is almost as strong as the twice-a-day (semi-diurnal) tide. Preliminary results demonstrate a strong concentration of turbulence at the crest of these two ridges. Interestingly, despite the strength of the diurnal tide, the turbulence at some locales is most strongly phased with the semi-diurnal forcing, both in terms of the location of the turbulence in the internal wavefront, and with the spring-neap cycle (Alford et al., 2011). Preliminary calculations indicate that this is because the two ridges are a distance apart
that promotes resonance of the semi-diurnal internal tide between the ridges, and dissonance for the diurnal frequency (Buijsman et al., 2012; Echeverri et al., 2011; Jan et al., 2007). This leads to strong semi-diurnal signals like those shown in Figure 4.

Two-dimensional Modeling

Observations of dissipation near supercritical topography are quite difficult to link to the generation mechanism. The turbulence varies by orders of magnitude over the tidal cycle, occupations must last at least a tidal cycle in order to get an accurate mean.
However, observations from only one location are hard to relate to a turbulence generation mechanism. To help with this interpretation problem, we have turned to numerical modeling at relatively high resolutions compared to most tidal models, but still coarse resolution compared to the turbulence (Klymak & Legg, 2010; Legg & Klymak, 2008). To date, we have simplified the problem by considering two-dimensional flows, but recent work is extending these efforts to three dimensions. By choosing an appropriate tidal forcing, stratification, and driving it over a two-dimensional topography, be it realistic or idealized, a picture emerges of the phenomenology of these breaking waves.

Figure 5: Simulated turbulence dissipation at a two-dimensional Hawaiian Ridge. Here the vertical resolution is 15 m, and the horizontal 150 m, and the surface tide forcing 0.06 m/s in the deep water, and 0.26 m/s over the ridge. The barotropic tide at ridge crest indicated with arrows and in text under the time relative to peak left-going flow. Large breaking waves, over 200 m tall, set up just after peak off-ridge flow, and then propagate across the ridge as the tide reverses direction. Note that the peak turbulence is after peak flow, as it takes a finite amount of time for the lee wave to grow.
**Phenomenology**

An example simulation (Figure 5) meant to represent flow over the Hawaiian Ridge at Kauai Channel, shows the lee waves we believe are the primary turbulence-producing phenomena active at super critical topography. During strong off-ridge flow (Figure 5b) a non-linear lee wave grows, and is arrested at the sill, reaching heights exceeding 200 m, and widths over a few kilometers (Figure 5c). As the tide relaxes, the wave is released and propagates on-ridge (Figure 5d). A similar structure forms on the other flank during the strong flow in the other direction. The shock-like character of this wave can be seen on its off-ridge edge, where the flow sharply rebounds analogously to the flow over an obstacle in a stream.

The simulation indicates why the site of observation can change what phase of the tide the turbulence can occur on. If observations are made downslope of the topographic break (say x=3 km, Figure 5), then the turbulence will arrive with off-slope flow. Conversely, if the observations are upslope (say x=1 km, Figure 5), then the turbulence will arrive during on-slope flow as the trapped wave propagates on-ridge. At a simple topography like the Hawaiian Ridge, this process is quite clear. At more complex bathymetry like the Oregon Slope (Figure 3), the phasing is probably very dependent on the exact placement of the observations relative to smaller scale changes in the bathymetry.

A fine-resolution simulation details what a typical lee wave looks like (Figure 6). The simulations are very reminiscent of observations in coastal fjords (Farmer & Armi, 1999; Klymak & Gregg, 2004) and simulations made in the atmosphere (Scinocca & Peltier, 1989), except that this motion only takes up 300 m of a 2000 m deep water column, and thus is a lot more “high-mode” than motions in those cases. A strong downslope flow accelerates moving down the slope, but density layers peel off sequentially with depth, driving structures that look like breaking hydraulic jumps. The remainder of the jet separates part way down the slope, at the depth where it hits ambient water, and rebounds with strong vertical oscillations, again reminiscent of a hydraulic jump.
Figure 6: Detailed simulation of the breaking lee wave during peak off-ridge flow (this time to the right). Here the vertical and horizontal resolution of the numerical model is 3 m, and the simulation is non-hydrostatic. Both panels have a one-to-one aspect ratio, and the crest of a smooth Gaussian topography is at 1000 m depth in a 2000-m deep water column. a) is temperature (proxy for density), where large 200-m breaking waves can be seen both in the downslope flow zone, and where the strong off-ridge flow separates from the topography. Note that the color has been shaded by the vertical derivative of the temperature in order to accentuate the turbulent structure of the flow. b) The cross-ridge velocity (red is off-ridge); the teal contours are temperature for comparison with (a). Again, the velocity has been shaded with the first difference in the vertical to accentuate turbulent structure.

Non-linear waves arrested at topography have a comprehensive literature for situations where the flow is strong and the obstacle relatively short (both compared to the strength of the stratification). Such flows tend to make waves that are “low mode” or large compared to the water depth (Baines, 1995). For deep-ocean ridges, the flow is weak compared to the height of the topography, and the vertical wavelength of the breaking waves quite short compared to the water depth. It can be shown, however, that the physics is the same (Klymak et al., 2010): if there is an off-ridge flow, the vertical mode with a horizontal phase speed close to that of the flow speed at the ridge crest is arrested. The size of this wave was predicted from internal wave theory, and compared to the size of waves generated in numerical simulations, with very good agreement. The time it takes for these waves to form depends on the topographic slope, with steeper slopes launching waves faster than gentle slopes. For an oscillating flow such as the tides, the finding is that the topography needs to be twice as steep as the internal wave
slopes for oscillating lee waves to be effectively trapped and dissipative (Klymak et al., 2010).

**Recipe for turbulence at supercritical topography**

Here we describe how the “arrested lee wave” concept allows for a prediction of tidal dissipation and mixing near supercritical topography based on the forcing, a simplified bathymetry, and the ocean’s stratification. First, a linear theory can be used to predict the internal response of tidal flow over sharp topography (Llewellyn Smith & Young, 2003; St. Laurent et al., 2003). The resulting flow field has very sharp “beams” of energy emanating from the ridge, and well-defined large-scale phase changes in the velocity on either side of these beams. This theory predicts an internal wave energy flux away from the topography, in each of the vertical modes (dashed line Figure 7d).

Non-linear numerical simulations of the same flow demonstrate similar features, particularly on the large scales (Figure 7b and c). However, the “beams” in the simulations are more diffuse, and some energy leaks out in higher-frequency (steeper) beams. As the forcing is increased (compare Figure 7b to c) the beams become even more diffuse. Diffuse beams indicate a loss of high-mode energy from the system, and this is quantified by noting that the energy flux at high modes does not agree with the linear theory (solid lines, Figure 7d and e). The stronger the forcing, the lower the mode where the disagreement starts, indicating a larger fraction of energy is dissipated.

To arrive at a dissipation rate, the recipe uses the observations above to determine the modes that dissipate. The strongest flows at the top of the topography are compared to the phase speed of the modes. Fast modes are assumed to escape and form the radiated signal; slow modes are assumed to be trapped and form the dissipation. The stronger the forcing, the more modes are trapped (Klymak et al., 2010). When compared to numerical simulations, this parameterization is quite successful over a range of tidal forcing, bathymetry shape, stratifications, and latitudes (the Coriolis parameter strongly affects the energy put into the waves), so long as the topography is sufficiently supercritical (Figure 7f).
Figure 7: a) snapshot of baroclinic (depth-mean removed) velocity from a linear generation from a 500-m tall knife-edge ridge in 2000 m of constant-stratification water, normalized by the barotropic forcing velocity. b) Snapshot of the same configuration when the tidal forcing is 0.04 m/s, and c) when it is 0.2 m/s. Note that the stronger forcing has less well-defined “beams” of energy radiating from the topography than the weaker, more linear, forcing. d) Comparison of the energy flux distributed by vertical mode number for a)-c) normalized by the energy flux of the barotropic wave. The 0.2 m/s case rolls off at lower mode numbers than the 0.04 m/s, as also shown in e). f) The parameterization of Klymak et al (2010) predicts this fraction of energy as a function of forcing, topographic height, and stratification, usually within a factor of 1.5.

An important finding of this analysis is that the local dissipation, while spectacular and locally important, is still a modest fraction of the energy removed from the surface tide. This can readily be discerned from Figure 1 or Figure 7, in which the full-water column motions that escape the ridges have significant energy. In fact, it requires very strong and non-linear forcing for the local dissipation to even approach 10% of the energy budget at these sills. Thus, supercritical ridges are believed to be quite efficient radiators of energy, at least in the idealized forms considered so far in the
modeling. A rough energy budget based on observations collected near Hawaii corroborates that idea (Klymak et al., 2006).

**More Complicated Systems**

Since supercritical topography seems to be efficient at generating internal tides without much local loss, the question stands as to the fate of that energy. Low-modes will impact remote topography where they will scatter, reflect, or dissipate. If that topography is supercritical, such as at other underwater ridges or continental shelves, the same physics discussed above appears to apply: cross-topography flow generates turbulent lee waves that are phase locked with the remote forcing, and a very similar parameterization is very effective at predicting the dissipation in these lee waves (Klymak et al., 2012 in review). Predicting the turbulence at a remote ridge can become quite complicated if there is local generation, and an incoming wavefield (Kelly & Nash, 2010). The phase between the local and remote forcing can change the dissipation predicted by an order of magnitude, as lee waves are either suppressed or enhanced by the resonance.

Nowhere is this effect more clear than in a two-ridge system like Luzon Strait. In two-dimensions, a two-ridge version of the linear model can be constructed that shows strong interactions between the two ridges. The two ridges must be treated as a system, and the relative heights of the ridges compared to the water depth, and the distance between them compared to the gravest internal mode wavelength, greatly affect the response. The potential for resonance means that a larger fraction of the internal tide might go into local turbulence than for isolated ridges. Recent estimates of the dissipation for Luzon Strait range from 40-20% (Alford et al., 2011; Buijsman et al., 2012).

**Discussion**

So far we have seen that the internal tide at abrupt topography often breaks, short circuiting the process of cascading internal wave energy that is often envisioned to fuel near-bottom turbulence over regions of small-scale topographic roughness (Polzin, 2009; St. Laurent et al., 2002). This local breaking is vigorous, producing turbulent events that reach hundreds of meters tall, and orders of magnitude greater than the open-ocean turbulence. The breaking is also phase locked to the tide, and strongly dependent on the
strength of the forcing. Numerical modeling indicates that a source of this turbulence at
many locations are lee waves arrested at the topographic breaks during off-ridge tidal
flow, either driven by the local tide, or by remote tides. We have a parameterization for
this process that works under a large range of forcing and topographies.

This progress in understanding the lee wave process does not preclude the
importance of other processes that perhaps do not manifest themselves in idealized two-
dimensional models. As indicated by (MacKinnon & Winters, 2005) and (Simmons,
2008), there is the potential for wave-wave interactions, particularly equatorward of the
critical latitude where the Coriolis frequency is half the tidal frequency. There is also
likely wave-wave interactions local to the Hawaiian Ridge, as indicated in observations
(Carter & Gregg, 2006), and as we have seen in our numerical models (Klymak et al.,
2010). However, it remains to be seen how important this energy pathway is near
topography, and there are indications that it is modest in observations of the radiated tide
(Alford et al., 2007).

There are a number of outstanding questions about the lee wave process, and
dissipation near supercritical topography. The first, is how good are estimates of the
turbulent dissipation in these processes? There have been very few direct estimates of
turbulence in these breaking waves; instead the size of the wave is compared to the
stratification it encompasses, and a dissipation rate inferred following (Thorpe, 1977).
Detailed comparison between microconductivity and the size of the breaking waves at
Hawaii had encouraging results (Klymak et al., 2008), as did comparisons between shear
probes and breaking waves in Knight Inlet (Klymak & Gregg, 2004), but direct
observations of turbulence dissipation at the microscale in these large mid-ocean lee
waves are still needed. Similarly, questions arise as to the efficiency of the mixing in
these breaking waves; i.e. what fraction of energy goes towards mixing density rather
than turbulent friction processes. Some of these questions will be answered by
forthcoming observations, particularly those in Luzon Strait. Others will require more
detailed modeling, likely at the scale of large-eddy simulations, though these will be
particularly challenging given the broad scale of the internal tides (100s of kms), and the
small eddy sizes (a few meters).
Much work to date, as described here, has been on two-dimensional idealizations of the flow. This is almost certainly an over simplification, and three-dimensional models of these problems are being run (Buijsman et al, personal communication).

Funneling of flow by constrictions can greatly enhance the lee waves, so deciding on an appropriate forcing and topography when applying the simple parameterizations discussed above is not trivial. Similarly, we have considered flow that is largely supercritical, except at the ridge crest. However, a lot of topography is more complicated than this, with significant roughness that can interact with the tides, or substantial near-critical regions that can very efficiently convert tidal flows into turbulence (Eriksen, 1982; McPhee & Kunze, 2002).

Despite these substantial caveats, we are attempting to create a global map of dissipation due to local dissipation at supercritical topography (indicated in Figure 8). There are estimates of tidal currents, stratification, and topography, so this problem should be tractable. To make progress, there are a number of challenging assumptions that need to be made when applying the parameterization suggested here globally: what scale should the topography be smoothed over to decide on its height? What tidal velocity should be used? How to account for the funneling of flows through constrictions? How to account for narrow topography that may not see substantial generation (Johnston & Merrifield, 2003)? Finally, if incoming internal tides from afar are important for local turbulence, either enhancing or suppressing it, then accurate maps of global internal tidal energy, perhaps at low modes, will be needed before a global estimate can be made (Kelly & Nash, 2010). This will prove challenging as tides are advected by large-scale currents and changes in stratification (Rainville & Pinkel, 2006), so it is possible global coarse internal tide models will need to be used to quantify this remote forcing (Simmons et al., 2004).
Figure 8: Regions of the global ocean with supercritical topography with respect to the semi-diurnal tide (colored red). This image is smoothed to half a degree for presentation, and some isolated supercritical regions exist elsewhere, but the major sources are indicated, i.e. the Hawaiian Ridge, Luzon Strait, Aleutian Ridges, the SW Pacific, and the continental slopes all show up prominently.

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