Tidally Generated Turbulence over the Knight Inlet Sill

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(Manuscript received 14 June 2002, in final form 18 November 2003)

ABSTRACT

Very high turbulent dissipation rates (above $\varepsilon = 10^{-2}$ W kg$^{-1}$) were observed in the nonlinear internal lee waves that form each tide over a sill in Knight Inlet, British Columbia. This turbulence was due to both shear instabilities and the jumplike adjustment of the wave to background flow conditions. Away from the sill, turbulent dissipation was significantly lower ($\varepsilon \approx 5 \times 10^{-7}$ to $5 \times 10^{-8}$ W kg$^{-1}$). Energy removed from the barotropic tide was estimated using a pair of tide gauges; a peak of 20 MW occurred during spring tide. Approximately two-thirds of the barotropic energy loss radiated away as internal waves, while the remaining one-third was lost to processes near the sill. Observed dissipation in the water column does not account for the near-sill losses, but energy lost to vortex shedding and near-bottom turbulence, though not measured, could be large enough to close the energy budget.

1. Introduction

Tides are an important source of energy in the oceans, both in the major basins and in marginal seas and estuaries. Tidal energy is dissipated directly by bottom friction or via form drag due to internal waves or vortical motions. Tidally generated internal waves are a possible source of energy that drives mixing in the deep abyssal ocean (Munk and Wunsch 1998). Vortical motions, created by flow separations around headlands, have been observed in coastal waters (Signell and Geyer 1991; Klymak and Gregg 2001), but their importance to energy budgets rarely considered.

Energy balances are difficult to calculate in the open ocean, whereas the confines of a coastal fjord are ideally suited to such studies. The barotropic tide in a deep fjord will typically behave like a nearly perfect standing wave; much of the barotropic tidal energy entering the fjord is reflected out again. The reflection, however, is never 100% efficient, and some energy is lost to bottom friction or internal motions. The energy budget of Knight Inlet, a fjord in central British Columbia, Canada, has been studied extensively. Freeland and Farmer (1980) calculated the energy lost from the barotropic tide between two pairs of tide gauges in the inlet and found that the majority of the energy was lost in a straight section containing a large sill. They hypothesized that the highly turbulent nonlinear lee waves near the sill (Farmer and Smith 1980; Farmer and Armi 1999a; Klymak and Gregg 2003) were responsible for dissipating most of the energy. Subsequent work by Stacey (1985) argued that the energy withdrawn from the tide can be accounted for using a linear internal wave model (Stigebrandt 1980). This model has not been tested with observational data and the application of a linear internal wave model is suspect.

This paper presents the first systematic measurements of turbulence in Knight Inlet, and one of the few examples of turbulence measurements in a fjord. Gargett (1985) presented data collected in Knight Inlet from a submarine, but that experiment was designed to test high Reynolds number turbulence rather than to constrain the energy near the sill. Here over 745 profiles were used to measure the dissipation of turbulent kinetic energy near the sill, with particular focus on the energetic nonlinear lee waves. The direct turbulent dissipation data was supplemented by another 3400 CTD casts from which the dissipation rate was estimated using the overturning method of Thorpe (1977).

After a brief description of Knight Inlet and the methods used to measure the flow and turbulence near the sill (section 2), we present detailed dissipation measurements (section 3). The detail is necessary because the strong temporal and spatial dependence of nonlinear flows do not average well, and some features were captured in very few events. Furthermore, we wish to justify
our use of the overturning method to estimate dissipation, increasing our statistics. An energy budget of the inlet is constructed by comparing the energy lost from the barotropic tide with energy radiated in internal waves and energy lost from the flow near the sill (section 4). A conclusion and discussion of the energy balance are offered in section 5.

2. Site and data collection

Knight Inlet is a fjord 300 km north of Vancouver, British Columbia, stretching 100 km into the mainland from the Queen Charlotte Strait (Fig. 1). The inlet is 2–3 km wide with vertical sidewalls. A sharp sill 25 km from the mouth divides the inlet into a seaward basin 160 m deep and a landward basin 450 m deep. A river at the head of the inlet provides a strong buoyancy source during summer when it is fed by glacier melt. The semidiurnal tide drives mean currents approaching $0.7 \text{ m s}^{-1}$ over the sill, generating strong nonlinear internal lee waves. These waves were the focus of a recent observational program that examined the time dependence (Farmer and Armi 1999a; Klymak and Gregg 2003) and the three-dimensionality of the flow (Klymak and Gregg 2001).

The data presented here were collected 18 August–3 September 1995 from the C.S.S. Vector, equipped with a 300-kHz RD Instruments (RDI) Broadband (BB) ADCP to measure the currents, a Guildline CTD to measure density, and a GPS for position determination, and from the R/V Miller equipped with a GPS and a 150-kHz RDI BB ADCP. ADCP performance on the Miller was suboptimal for most of the cruise; noise was not averaged out satisfactorily. Two profiling packages were deployed from the Miller. The first, the Shallow Water Integrated Mapping System (SWIMS), is a towed body equipped with two SeaBird 911 CTDs designed to point into clear water while being towed up and down diagonally through the water as the ship steamed at 1 m s$^{-1}$. The second, the Advanced Microstructure Profiler (AMP), is a loosely tethered profiler capable of CTD and standard microstructure measurements [see Wesson and Gregg (1994) for details about this instrument]. Near the sill, SWIMS, AMP, and Vector CTD casts were made east–west along the inlet axis, except for one day during flood tide when the R/V Miller ran a series of cross-inlet lines (Fig. 2). Note that the deepest part of the inlet on the landward side of the sill is actually slightly south of the along-inlet lines. Both the R/V Miller and C.S.S. Vector were equipped with Biosonics 120-kHz echo sounders, capable of resolving backscatter from small biology and microstructure in the water column (Seim et al. 1995). Farmer and Armi (1999a) present a number of these echo-sounder images from the C.S.S. Vector.

We also deployed two SeaBird tide gauges for the duration of the cruise, one 9.8 km west of the sill and the other 2.5 km east. These gauges measured the pressure at hourly intervals using a 60-s-long burst sampled at 1 Hz and then averaged to give an accuracy of 0.5 cm. Tidal heights were estimated by removing the cruise-mean pressure signal; a harmonic analysis (discussed below) shows a phase lag between the two gauges due to energy withdrawn from the barotropic tide.

The primary method for estimating turbulence was our loosely tethered dissipation profiler AMP The AMP and how it is used to calculate turbulent dissipation are described in Moum et al. (1995) and Wesson and Gregg (1994). Briefly, it measures small-scale velocity fluctuations as it falls through the water by measuring the change of the flow direction on a small airfoil. A spectrum of turbulent shear can be estimated if the velocity of water past the probe ($w$), the probe’s sensitivity, and
its response to velocity fluctuations are known. The shear spectrum is then integrated to give the turbulent dissipation rate $\varepsilon$, presented here in units of watts per kilogram. The only measurement difficulty was estimating the speed of the flow past the profiler. This is usually done by differentiating the pressure signal, but this at times required correction to account for the very strong vertical velocities observed in Knight Inlet. The turbulent dissipation rate depends inversely on the magnitude of this velocity to the fourth power ($\varepsilon \propto w^{-4}$), and this correction is significant for a number of drops in the lee wave (see the appendix).

Few AMP casts were made, and their temporal resolution was poor in comparison with tow-yoed CTDs. Turbulent dissipation rates were inferred from CTD data by calculating the size of density overturns $L_t$ (Thorpe scale) and using the Ozmidov scaling:

$$\varepsilon_{\text{inferred}} = \alpha N^4 [\text{rms}(L_t)]^2,$$

where $\alpha \approx 1$ (Thorpe 1977) and $N$ is the buoyancy frequency. There is excellent agreement between the Ozmidov scale and the Thorpe scale, though there is usually a scatter of a factor of 4 for individual overturns (Dillon 1982; Moum 1996). Wesson and Gregg (1994) and Seim and Gregg (1994) present data from similar high-energy regimes (Strait of Gibraltar and Admiralty Inlet) and find that the overturn method is applicable. Large energy dissipation events were detected easily by the overturning method, except in the upper 10 m of Knight Inlet where the high stratification made overturns smaller than the resolution of the CTDs.

3. Observations

Here both SWIMS and AMP data from similar phases of the tide are compared to estimate how well SWIMS captured turbulent events. Then the importance of cross-channel structure to the turbulence is discussed. Last, data collected upinlet of the sill are presented.

Turbulence was averaged from all near-sill data binned by tidal phase and space (Fig. 3). The turbulent diffusivity $K_r = 0.2 \varepsilon / N^2$ was high everywhere near the sill ($K_r \geq 10^{-2}$ m$^2$ s$^{-1}$). However, there were two thin regions on the seaward (at -200 m) and landward sides (at 1500 m) of the sill where the average diffusivity ($K_r$) $> 10^{-1}$ m$^2$ s$^{-1}$. The structures that cause these high diffusivities are discussed in detail below. They are very time dependent and are forced by the response of the stratified fluid to the tide. Here, we present a brief description of the time dependence and then describe the flows that went into the average in Fig. 3 in detail during the rest of the section.

a. Time dependence

The time-dependent flow over the Knight Inlet sill is quite complicated and has been discussed in detail by Farmer and Armi (1999a), Armi and Farmer (2002), and Klymak and Gregg (2003). An example of data from a weak ebb tide, a strong ebb tide, and a strong flood tide are shown in Fig. 4. During each tide a nonlinear lee wave formed downstream of the sill. The response was analogous to hydraulic control of a two-layer fluid, described in detail by Baines (1984, 1995: see Fig. 5.28 and p. 296). Under a wide range of conditions, water upstream of the obstacle does not have enough energy to surmount the obstacle, and so the lower layer is partially blocked. Water accumulates in the dense layer lifting the interface upstream, while there is a divergence downstream dropping the interface there. A nonsteady situation persists near the sill until enough potential en-
Fig. 4. The flow over the sill in the hours leading up to the data presented below. (a)–(c) A weak ebb tide, (e)–(g) a strong ebb tide, and (h)–(i) a strong flood tide. In each panel, velocity is colored, and density is contoured at intervals \( \sigma = 16, 21, 23, 24, \) and 24.5 kg m\(^{-3} \). The small ship on top of each plot represents the direction in which the Miller was traveling. Notation on top of each panel is “t” for hours past high tide and “Q” for volume flux \( (10^3 \text{ m}^3 \text{s}^{-1}) \). The clocks also represent hours past high tide. Detailed plots of the data for (c), (f), and (i) are given below.

Energy is stored in the lower layer to drive all of the lower layer over the obstacle at a fast enough rate. Once in steady state the flow at the obstacle crest is hydraulically controlled in a manner analogous to water passing over a weir, and the two layers meet a criticality criterion of 
\[ G^2 = u_i g h_1 + u_s g h_2 = 1 \] (Armi 1986). The lift of the interface upstream and the drop downstream propagate away from the obstacle as waves, raising the energy of the water upstream and lowering it downstream. Water flowing over the obstacle must dissipate energy for the upstream and downstream conditions to match. In a two-layer (or one layer) fluid much of this dissipation takes place through an abrupt hydraulic jump.

Many features analogous to the two-layer case can be seen in Fig. 4 and in Klymak and Gregg (2003). The isopycnals have been raised upstream and depressed downstream of the sill and form waves that radiate away from the sill. There is a plunging flow over the sill and an abrupt rebound of isopycnals to the background state. There are also differences with the simple two-layer analogy. Farmer and Armi (1999a) and Armi and Farmer (2002) show that a critical Froude number can be found near the sill crest along the strong shear interface between the plunging flow and the flow aloft. However, the stratified interface does not follow a simple hydraulic solution (Farmer and Armi 1999a) as isopycnals peel off from the interface successively. Instead of a localized hydraulic jump, there is a large region in the lee of the sill where the flow adjusts to the downstream state (Fig. 4i); we will show below that this region dis-
sipates considerable energy. There are also other ways to dissipate the energy required of the hydraulic response. As we will see below, and was also pointed out by Farmer and Armi (1999a), the flow develops shear instabilities that are quite vigorous. There is also the possibility of strong bottom friction (Nash and Munk 2001) and lee vortices (Klymak and Gregg 2001; MacCreary and Pawlak 2001). These sinks of energy will all be quantified in section 4 and demonstrated by example below.

b. Weak ebb tide

Our first example shows the response over the sill during a weak ebb tide (Fig. 5). There were two shear regions in this flow. The first shear region was along the bottom of the lee wave starting near 10-m depth 1 km landward of the sill crest and extending downstream; it dropped, crossing isopycnals until −0.5 km (Fig. 5a). This shear interface was populated with flow instabilities evident in the strong backscatter that began near the surface and then became braided as the shear interface deepened over the sill crest (Fig. 5b). These billows had dissipation rates approaching $\varepsilon = 10^{-4}$ W kg$^{-1}$ (Fig. 5c).

A second turbulent region was observed in the flow separation near the sill crest. The turbulence began at the sill crest and extended west to −0.75 km at about 80-m depth. This interface was not detected clearly by the echo sounder (Fig. 5b), presumably because the stratification was much weaker at these depths. Careful examination of the echo-sounder data showed signs of breaking waves (see Fig. 6 for larger images); previous studies in Knight Inlet have shown large instabilities near this location (Farmer and Smith 1980; Farmer and Armi 1999a; Klymak and Gregg 2003). The region was populated by overturns, but the implied turbulence, $3 \times 10^{-6}$ W kg$^{-1}$, is weaker than in the wave shear layer (Fig. 5c). The water beneath the separation was stably stratified and not turbulent.

There was little turbulence observed in the rest of the water column. Upstream of the sill crest the water was quiescent, as it was downstream below the wave shear layer. The turbulence in the wave shear layer was localized to the shear layer, dropping off quickly above and below. Sampling with SWIMS did not reach closer than 5 m from the bottom, and so turbulent overturns in the bottom boundary layer may have been missed. These data are in agreement with those collected in Amp drops through very similar flows the previous day. Figure 6a presents four drops through a widening wave. Drops 15950–15952 show that the turbulence was localized to the shear layer beneath the lee wave, starting at about 17-m depth at 0.4 km and then dropping to 25-m depth by 0 km. The turbulence reaches $\varepsilon = 10^{-4}$ W kg$^{-1}$ in this interface, is very low below the interface ($\varepsilon = 10^{-8}$ W kg$^{-1}$), and is moderate above the interface ($\varepsilon = 10^{-6}$ W kg$^{-1}$). Data in drop 15951 indicate a lower dissipation rate in the wave shear layer farther upstream. Turbulence decreased downstream; at drop 15953 the dissipation at 30 m was only $\varepsilon = 10^{-8}$ W kg$^{-1}$. High turbulence was measured at 60–80 m in drop 15953 (−0.2 km) as the instrument traveled through the flow-separation region, but decreased again below. The echo-sounder image indicated billows along the flow-separation interface.

In a pass over the sill made 30 min later (Fig. 6b) the shear interface had deepened. The interface also moved to denser isopycnals (not shown). The turbulence along this interface was again quite high ($\varepsilon = 10^{-4}$ W kg$^{-1}$) but low above and below. Figure 6 emphasizes the time dependence of the changes in the flow.

c. Strong ebb tide

The beginning of a strong ebb tide looked much like a weak ebb tide. As the tide progressed, however, the response grew stronger (Fig. 7). A wave shear interface with the billows remained evident, but, as it approached the sill crest, it plunged more sharply. This interface rebounded at about 100-m depth and demarks an undular wave in the lee of the sill.

The wave shear layer was populated by shear instabilities that were larger with depth (Fig. 7b). The dissipation rate of these overturns was $\varepsilon = 10^{-4}$ W kg$^{-1}$. The large undulation on the seaward side of the lee wave also has a dissipation rate above $\varepsilon = 10^{-4}$ W kg$^{-1}$. This feature was not an overturn, but rather a very steep wave that appeared to lose energy soon downstream of the first undulation. As such, we will treat the height of the undulation as the vertical scale instead of the proper Thorpe scale in Eq. (1). The turbulence associated with this undulation extended up in the water column to 25-m depth but dropped quickly farther seaward, reaching levels consistently below $\varepsilon = 10^{-6}$ W kg$^{-1}$ by −0.5 km.

Again, the SWIMS data agree well with the direct turbulence measurements from Amp (Fig. 8). Upstream and downstream of the undulation on the seaward side of the lee wave the turbulence was low except in the shear interface near 15-m depth (drops 15769 and 15770). In the undular wave (drop 15771) high dissipation was measured almost to the top of the water column; values were slightly above $\varepsilon = 10^{-4}$ W kg$^{-1}$. The turbulence measured in the next drop (15772) at −0.6 km was still elevated but not as high, generally around $\varepsilon = 10^{-5}$ W kg$^{-1}$, and drop 15773, just seaward, was below $\varepsilon = 10^{-6}$ W kg$^{-1}$ (not shown).

d. Flood tide

Flood tide was moderately strong and did not differ as much from neap to spring (Fig. 9). A large lee wave formed early in the tide with the characteristic elevated isopycnals upstream, plunging flow over the sill crest, and a rebound to downstream conditions by 1.5 km east
of the sill (Figs. 4g–i). The turbulence appeared mostly when isopycnals in the lee wave expanded, which they did at progressively deeper depths downstream. The strongest overturns occurred where the distance between isopycnals widened in the lee of the sill and individual isopycnals became vertical (Fig. 9c). Some of the turbulence was perhaps also caused by shear instability; in particular, the turbulence between 1 and 1.5 km was associated with billows in the echo-sounder image between 25- and 55-m depth. During flood it was more difficult to separate the two processes definitively, partly because the flow was more three-dimensional than during ebb and our sampling sliced the flow at an awkward angle (see below). The dissipation was low over the plateau of the sill until 1 km when it increased. By 2 km it had returned to a low level.

Similar dissipations were found in AMP data collected during flood tide (Fig. 10). Upstream, in drops 15919–15916, the turbulence was low and confined to a thin shear layer above 20-m depth. Downstream, as the sill steepens, the turbulence was high (drop 15915), especially between 25 and 50 m. This region was populated by shear instabilities and nearly vertical isopycnals, and so, again, saying whether the turbulence was...
from shear or breaking of the lee wave is difficult. Further downstream (drop 15914), the turbulence below 75 m was not clearly associated with features in the echo sounder, suggesting the importance of breaking waves. By 2.1 km (drop 15913) there was almost no turbulence in the flow.

e. Three-dimensionality of the turbulence during flood

During flood tide there was large cross-inlet variability in the dissipation, as expected from the large cross-inlet variability of the flow (Klymak and Gregg 2001). On August 26 two cross-inlet lines were repeatedly occupied, one at the sill crest and the second 1.5 km downstream (Fig. 11). There was an hourglass-shaped jet of landward-flowing water at middepth flanked by two counterflowing jets (discussed in detail by Klymak and Gregg (2001)). At middepth, the landward jet was less than 300 m wide, creating a complicated pattern of shear in the flow. The shear lines between the landward jet and the counterflows were populated by increased backscatter showing undulations with 25-m amplitudes (i.e., at −0.4 km and 50-m depth; Fig. 11b). These undulations appear as large overturns in the SWIMS CTD data implying dissipation rates close to \( \varepsilon = 10^{-4} \) W kg\(^{-1} \) centered on the landward jet (Fig. 11c). The rapid return flow on the south side of the channel also appeared turbulent, but the slow return flow on the north side was mostly quiescent.

f. Upinlet turbulence

Sampling was concentrated near the sill, but some data were collected far enough away from the sill that an average profile of turbulent dissipation can be formed. Considering all drops made 6 km or more away from the sill, the dissipation rate was modest (Fig. 12). The turbulence profile was generally as high as \( \varepsilon = 10^{-7} \) W kg\(^{-1} \) in the upper water column, and \( \varepsilon = 10^{-8} \) W kg\(^{-1} \) at 100-m depth. Below 100 m it was \( 10^{-9} < \varepsilon < 10^{-8} \) W kg\(^{-1} \), though the small number of samples below 100 m may have biased this number.

A larger ensemble of profiles made upinlet were collected while following nonlinear internal wave packets. In order to not bias the average, these profiles were not included in the average presented. These packets manifested themselves as a series of rapid undulations in the near-surface pycnocline. These “solibores” had dissipations reaching \( \varepsilon = 10^{-5} \) W kg\(^{-1} \) in limited patches, approximately 1000 m × 10 m (Fig. 13). The structure of this example was not dissimilar to the lee waves, with a turbulent shear layer along a bottom interface.
4. Energy budget

The energy that feeds the internal motions in the inlet mainly comes from the barotropic tide. Freeland and Farmer (1980) discount the importance of the wind, which seems reasonable during our mostly windless cruise [see Farmer and Armi (1999b) for an aerial photograph of glassy water]. The energy budget has already been discussed in detail by Freeland and Farmer (1980) and Stacey (1985), who found using tide gauge data that most energy from the barotropic tide is lost near the sill. Where does this energy go? Stacey (1985) used a linear tidal model to argue that much of it went to creating internal tides but did not have observational evidence to verify it. Freeland and Farmer (1980) cite the large nonlinear lee waves observed by Farmer and Smith (1980) as another possible sink but had no means to estimate the energy dissipated by these structures.

Here we determine how much energy is lost from the barotropic tide and then estimate possible sinks (sketched in Fig. 14). Sinks include radiating internal waves ($P_{SW}$ and $P_{LW}$), dissipation in the breaking lee waves ($D_{I}$), losses to bottom friction ($D_{F}$), and creation of vortical structures ($D_{V}$). All of these sinks must eventually dissipate energy through turbulence, but the na-
tecture of the sink affects where the dissipation will take place and whether it was captured by the observations presented above.

a. Energy lost from the barotropic tide

The energy lost from the barotropic tide can be estimated by considering the phase difference between two tide gauges (Freeland and Farmer 1980). If no energy is lost, the barotropic tide in an enclosed inlet will be a perfect standing wave. If energy is lost, the reflected wave will be weaker than the incoming wave, and the seaward tide gauge will lead the landward one. If the phase lag is small, Stacey (1984) derived that the power lost from the barotropic tide between two tide gauges can be estimated from the phase lag between them, $\phi_1$, as

$$P_T = \frac{\rho g \eta_0^2 \omega S_1}{2} \left( 1 - \frac{S_0}{2S_1} \right) \phi_1,$$

where $\eta_0$ is the height amplitude of the barotropic tide (m), $\omega$ is its frequency (rad $s^{-1}$), $S_1 = 2.4 \times 10^8$ m$^2$ is the surface area upinlet of the seaward tide gauge, and $S_0 = 3.6 \times 10^7$ m$^2$ is the surface area between the two tide gauges.

Here a harmonic analysis at two frequencies, semidiurnal (12.42 h) and diurnal (23.93 h), estimates the phase difference between the two tide gauges. The analysis was carried out over a 1-day sliding window. The phase lag at the semidiurnal frequency between the two tide gauges was approximately constant at $\Delta \Phi = 1.8^\circ$ (Fig. 15c); the phase lag at the diurnal frequency was small. The semidiurnal amplitude varied from 0.9 to 2 m over the course of the cruise (Fig. 15b). Therefore the power lost from the semidiurnal tide according to Eq. (2) varied from 4 MW at neap tide (day 231, Fig. 15d) to 20 MW at spring (day 239).

b. Energy dissipated near the sill

To estimate the energy sinks near the sill, we considered data that covered the length of the sill and allowed good estimates of the turbulence and waves in the flow. To meet these requirements four days of data collected using SWIMS were used—two during moderate tides, one during a spring tide, and one during a neap. Data are from 800 m seaward to 2000 m landward of the sill. We compare the energy lost from the barotropic tide $P_T$, waves radiated seaward from the sill $P_{SW}^W$, waves radiated landward of the sill $P_{SW}^L$, and energy sinks near the sill, $P_B$:

$$P_T \sim P_{SW}^W + P_{SW}^L + P_B.$$

The term $P_B$ is energy lost near the sill that does not manifest itself as internal waves. This energy drop is then compared with the dissipative terms:

$$P_B \sim D_e + D_f + D_v,$$

where $D_e$ is dissipation observed in the interior of the flow, $D_f$ is dissipation inferred along the bottom, and $D_v$ is the rate that energy is transferred to headland eddies. The strength of these terms is plotted for the four days of SWIMS data (Fig. 16) and discussed in the remainder of this section.

c. Wave flux away from the sill

The internal wave flux from the sill is estimated using two direct methods and then compared with a linear theory. The internal waves are the manifestation of the upstream and downstream propagating disturbances that are part of the response of the flow to the sill (section 3a). To estimate the wave flux we first consider the correlation between velocity and pressure perturbations:

$$P_{W(1)} = \Delta y \int_{-D}^{0} u'p' dz,$$
where $\Delta y$ is the width of the inlet at the seaward and landward sections (Kunze et al. 2002). The velocity perturbations are computed by removing the barotropic velocity expected from the tide gauge data. The pressure perturbation is calculated by assuming hydrostatic pressure

$$p' = g \int_0^z \rho' \, dz,$$

where $\rho' = \rho - \rho_0$ is the density perturbation at a given depth. Throughout, $\rho_0$ is calculated separately on either side of the sill for each set of SWIMS runs by averaging two density profiles bracketing the slack tide.

This calculation yields the light gray traces in Figs. 16e–h and 16m–p. The direction for the seaward fluxes was to the west, but they have been plotted as positive. The flux estimate is not likely to work on the landward side of the sill during flood tides where the velocity is strongly influenced by horizontal vortices (see Fig. 11 and Klymak and Gregg 2001). On the seaward side of the sill, $\int u'p' \, dz$ varies with the square of the transport and reaches as high as 14 MW during flood tide on day 236 (Fig. 16f).

Wave fluxes are also estimated by calculating the amplitude of the internal waves using a modal decomposition. Wave motions are decomposed into the three lowest vertical modes. The total energy flux is
Fig. 10. Dissipation measured using AMP in the lee wave during flood tide, 2224–2316 UTC 1 Sep 1995.

Fig. 11. A cross-channel section of SWIMS data from 1.5 km landward of the sill (1846 UTC 24 Aug 1995): (a) upinlet velocity, (b) acoustic backscatter, and (c) turbulent dissipation.

Fig. 12. Average turbulent dissipation profile for all data taken more than 6 km landward of the sill. Data density is indicated by shading. Drops shorter than 70 m were excluded from the average because they preferentially followed nonlinear wave packets. There were only 10 profiles that extended below 100 m. Bootstrap estimates of the dissipation error are plotted as thin lines around the mean.

The amplitude of each mode, \( \xi_n \), is calculated by regressing the displacement with the standing vertical mode structures. This expression for the energy density is an approximation assuming that energy is equipartitioned between potential and kinetic energies. We tested this assumption by calculating the kinetic energy flux on the seaward side of the sill where the flow is two-
dimensional and found that it agreed well with the potential energy flux, though the results are not plotted for clarity.

The modal estimate, Eq. (7) (black line in Figs. 16e–h, m–p), gives similar results to the \( u'p' \) estimates on the seaward side of the sill, but not on the landward where Eq. (5) is likely inaccurate. The flux tends to be 15% higher on the landward side of the sill, yielding an energy flux that peaks at 17 MW on day 236.

These direct estimates are compared with the linear estimate developed by Stacey (1985) and Stigebrandt (1980). This method uses the vertical modes and assumes continuity of the barotropic velocity at the sill crest \( (U_s) \):

\[
P_{\text{lin}}^{\text{dir}} = \rho \Delta y U_s^3 \sum_{n=1}^{3} \frac{c_n \bar{W}_n(z_s)}{2 \int_{-D}^{0} (dW_n/dz)^2 \, dz},
\]

where \( z_s \) is the sill depth. This linear estimate gives the dashed curves in Figs. 16e–h and 16m–p. There is a relatively good agreement in energy flux between the methods, though the linear estimate tends to be smaller.

There is also a phase difference between the direct and linear estimates (and thus the direct estimates and the barotropic velocity at the sill); the direct estimates lead the forcing on the downstream side of the sill and lag it on the upstream side.
As discussed above (section 3a), the waves that radiate from the sill at each phase of the tide raise the energy upstream and lower it downstream. This means that water flowing over the sill must dissipate energy to match the upstream and downstream conditions. The dissipation of energy in a steady-state flow can be estimated from the divergence of flux of the Bernoulli function:

$$P_B = \int_V \rho e \, dV = \int_A \rho B \mathbf{u} \cdot dA, \quad (10)$$

where the Bernoulli function is $B = \frac{1}{2} \mathbf{u}^2 + P/\rho + gz$ (Baines 1995, p. 13). Nash and Moum (2001) used a one-layer approximation of this equation for an energy balance over Stonewall Bank. To estimate the right-hand side of Eq. (10), we divide the flow into five layers (divided at $\sigma_x = 0, 21, 22, 23, 24, \text{and} \ 25 \text{ kg m}^{-1}$) and calculate the Bernoulli function in each layer at the section downstream of the sill $[B^o]$ and the section upstream $[B^{uo}]$. The volume flux $q_n$ is assumed constant in each layer between the two sections; that is, the mixing in the flow is small. Power lost is

$$P_B = \sum_{n=1}^{5} q_n [B^{uo} - B^o]. \quad (11)$$

The constant flux assumption is necessary because the lee vortices preclude a good estimate of downstream volume flux in each layer. The surface pressure is also not known precisely, but is estimated to be strong enough to accelerate the flow so that the volume flux remains constant in the surface layer; this correction corresponds to 0.01 m, or approximately 10% of the Bernoulli drop in the deeper layers.

The resulting energy drop (black line in Figs. 16i–l) shows spring–neap modulation and a maximum energy drop similar to the observed wave flux (~15 MW). Interestingly, neap tides (Figs. 16i and 16l) have disproportionately small Bernoulli drops.

These two-dimensional estimates of the Bernoulli drop can be compared with a similar estimate made using the cross-channel sections (Klymak and Gregg 2003). We found a 16-MW drop between two cross-channel sections at peak flood tide, similar to the value in Fig. 16j. We did not exclude mixing in the three-dimensional calculation, and so the agreement indicates that our assumption of two-dimensionality does not bias the estimate unduly.

e. Energy sinks near the sill

The Bernoulli drop must be caused by dissipation or storage of energy between our two sections or by three-dimensional motions. There is very large dissipation in the shear and convective instabilities near the sill. However, when averaged, this turbulence is not large enough to account for the Bernoulli drop. Dissipation is estimated from the measurements made during the SWIMS runs using the overturn method and summing:

$$P_x = \Delta y \int_{-D}^{D} \int_{-800}^{2000} \rho e \, dz \, dx, \quad (12)$$

where $D$ is the depth at each SWIMS cast. The width, $\Delta y$, is the width over which the dissipation measured during the SWIMS run is representative. Assuming that the turbulence is confined to the region where there are large lee waves and not behind the headlands, we choose $\Delta y = 1200$ m. The results (dotted line, Figs. 16i–l) suggest that the water column dissipation is too small to account for the energy drop across the sill. Even for a run that passed through the highest dissipations during flood tide (Fig. 16k and Fig. 9 above) the dissipation only reaches 50% of the energy drop across the sill, peaking at 5 MW.

There are probably other sinks of energy not captured by SWIMS. First, there could be large sinks of energy
near the bottom where we did not measure with AMP or SWIMS. To estimate the capacity of this energy sink, we assume a cubic dependence:

\[ P_v = \int k\rho|u|^3 \, dA, \]  

(13)

where \( k = 0.003 \) (Freeland and Farmer 1980). This yields an estimate (dashed–dotted lines in Figs. 16i–l) that is approximately as large as the mid water column turbulence. If there were a turbulent region 5 m thick along the bottom, then dissipation rates in it would need to reach \( \varepsilon = 5 \times 10^{-3} \) W kg\(^{-1}\). This is a high, but not unreasonable value. Nash and Moum (2001) observed bottom boundary layer turbulence of \( \varepsilon = 10^{-2} \) W kg\(^{-1}\) for flow over a shallower obstacle on the continental shelf.

The direct dissipation measurements and inferred bottom dissipation remain inadequate to account for the drop in the Bernoulli flux across the sill. There is a storage of potential energy in the lee waves. When estimated from the sections it is found to be much smaller than the other terms. For instance, a 2000-m-long interface displaced at 12.5 m h\(^{-1}\) with a density difference across it of \( \Delta \rho = 1 \) kg m\(^{-3}\) would have a rate of change of available potential energy \( \frac{dPE}{dt} = \Delta y \Delta x \eta^2 \Delta \rho / \Delta t = 0.6 \) MW. This is not a small accumulation of energy, but it is accounted for eventually in either the wave radiation or in the dissipation. In fact, the release of these waves may be responsible for the phase differences observed between the linear calculation and the wave flux estimates (Fig. 16).

A final energy sink is the lee vortices shed during each tide. Vortical motions do not show up in the wave flux estimates despite containing significant energy that can advect away from the sill region or dissipate on sidewalls. During peak flood tide the vortices have a kinetic energy of \( 2.3 \times 10^7 \) J m\(^{-1}\) (Fig. 11). Multiplying...
this by the distance to the sill (1.6 km) gives $3.7 \times 10^{16}$ J, which, distributed over the three hours of flood, gives a rate of 3.4 MW of energy being fed into the vortices. This only accounts for energy in the vortices toward the sill; they also extend an unknown distance downstream.

A simple parameterization of the vortex strength gives a time dependence to this energy sink. For flow separation around bluff bodies, the drag coefficient can be considered to be of order unity ($C_d \approx 1$). Recent numerical experiments by MacCready and Pawlak (2001) indicate that this drag coefficient is appropriate for a ridge protruding into flow along a slope. The energy lost from a flow with velocity $U$ impinging on an obstacle with a cross-sectional area $A$ is given by $P_D = 0.5C_u U^3 A$. For the pair of headlands we estimate an area of $2 \times 500 \text{ m} \times 60 \text{ m}$ to calculate energy removed from the mean flow (dotted line, Figs. 16i–l). The energy removed from the vortices dominates the other dissipative terms, peaking at 7 MW (Fig. 16j). In comparison with the estimate of 3.4 MW above, the vortices would need to be 3.2 km long at flood tide, which is not an unreasonable length. Klymak and Gregg (2001) show that the vorticity in each vortex between the sill and the landward section is $3400 \text{ kg m}^{-1} \text{s}^{-1}$. Vorticity production at each headland was estimated at a rate of $1.5 \text{ kg m}^{-1} \text{s}^{-2}$, and so a vortex with $3400 \text{ kg m}^{-1} \text{s}^{-1}$ of potential vorticity could be produced in less than 0.75 h, and one 2 times as large in 1.5 h.

The energy put into the lee vortices can be compared with an estimate for the energy lost from a barotropic tide at a constriction (Stigebrandt and Aure 1989):

$$E_j = 0.42(1/4)\rho U_s^3 A,$$

where $A_s = 9 \times 10^4 \text{ m}^2$ is the area at the sill cross section and $U_s$ is the amplitude of the barotropic velocity. For spring tides barotropic velocities reach 0.6 m s$^{-1}$ over the sill, leading to $E_j = 2 \text{ MW}$. This is in good agreement with the spring-tide estimate of energy put into the lee vortices.

5. Summary and discussion

Turbulent dissipation rates near the Knight Inlet sill were examined and put into the context of the inlet’s total energy budget (Table 1). Dissipation rates reaching $\varepsilon = 10^{-4} \text{ W kg}^{-1}$ were observed near the sill associated with shear instabilities and breaking in the lee waves. The integrated dissipation, however, was always much smaller than the energy removed from the barotropic tide.

The energy removed from the barotropic tide can be accounted for by energy radiated in internal waves ($P_\text{IW}^{\text{SW}}$ and $P_\text{IW}^{\text{LW}}$) and energy losses near the sill ($P_s$). Wave energy radiated landward was always larger than wave energy radiated seaward and both were approximately the same as the nonradiated Bernoulli drop, and so approximately two-thirds of the energy was radiated and one-third was dissipated locally. After the diurnal inequality is accounted for, the energy sinks balanced the energy lost from the barotropic tide within 20%.

<table>
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<th>Day</th>
<th>$P_T$</th>
<th>$P_B$</th>
<th>$P_\text{IW}^{\text{SW}}$</th>
<th>$P_\text{IW}^{\text{LW}}$</th>
<th>$P_s$</th>
<th>$D_s$</th>
<th>$D_f$</th>
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<tr>
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<td>0.2</td>
<td>0.1</td>
<td>0.5</td>
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</table>

$^1$ The diurnal inequality is not resolved by the harmonic analysis used to calculate $P_T$; attempts to perform the analysis on windows shorter than a day were too noisy. The correction is to assume that the energy removed is quadratic with the velocity at the sill so that $\langle \varepsilon \rangle = \langle P_T \rangle \langle u'^2 \rangle / \langle u'' \rangle$, where $\langle u'' \rangle$ has been smoothed over a day to match $P_T$. The angled brackets represent averages over the same time period as the data in the previous sections.
where they dissipate because of self-interaction or sidewall friction. The vortices are also baroclinic, and vortex stretching appears to spin up the mid water column more than the water above and below (Klymak and Gregg 2001). We do not know how the baroclinicity changes the dynamics of the dipole.

The energy sinks near the sill, however estimated, are about one-third the total energy lost from the barotropic tide. Therefore radiating internal waves are important, as hypothesized by Stacey (1985). Recent studies indicate that this is also the case at obstacles in the open ocean. For instance, approximately 5% of the energy removed from the barotropic tide at the Mendocino Escarpment goes into local turbulence, the remainder radiates away as internal waves (Althaus et al. 2003). Where and how this internal wave energy ultimately dissipates remain an important open question. In Knight Inlet, the observed upinlet dissipation is too small (Fig. 12), as are the nonlinear bores (Fig. 13). Transmission at the bend (Fig. 1) or the head of the inlet is likely inefficient.

Acknowledgments. We thank those whose expertise made the collection of these data possible: Jack Miller, Earl Krause, Steve Bayer, and the master of the R/V Miller, Eric Boget. David Farmer and Eric D’Asaro kindly provided data. Discussions with David Farmer, Eric D’Asaro, Jonathan Nash, Eric Kunze, Matthew Alford, Jennifer MacKinnon, Michael Stacey, and Stephen Pond were all very useful. The comments of two anonymous reviewers greatly improved the manuscript. We are grateful to the U.S. Office of Naval Research for financial support under budget numbers N00014-95-1-0012 and N00014-97-1-1053, and the SECNAV/NCO Chair in Oceanography held by MG.

APPENDIX

Flow past AMP versus Fall Rate

Estimates of turbulent dissipation depend inversely on $w^4$, the speed of water past the shear sensor, and so a poor estimate of this speed can drastically alter the estimate of dissipation. Standard procedure is to estimate the flow speed past the probe using the fall rate of the instrument, easily derived from the pressure $P$:

$$w = -10^{-4}(\text{Pa m}^{-1}) \frac{dP}{dt}. \quad (A1)$$

However, there are rare occurrences in which vertical velocities reach $\pm 0.4 \text{ m s}^{-1}$ in Knight Inlet, and so the fall rate calculated from the pressure is no longer an accurate representation of the flow rate past the probe. Most of the time, instruments were set to fall at $0.5 \text{ m s}^{-1}$, and so $0.4 \text{ m s}^{-1}$ counterflow means an estimated fall speed of $0.1 \text{ m s}^{-1}$; coupled with a fourth-power dependence on $w$, this leads to a catastrophic error in the dissipation estimate.

This is corrected by assuming that the instrument falls at a predictable rate. The buoyancy of the instrument can be adjusted to vary the fall rate. Three different fall rates were used during the cruise: 0.25, 0.5, or 0.75 m s$^{-1}$ (Fig. A1). In quiet water, the instrument tended to slow down during its decent. The scatter in fall rates is not too great, except in a few cases in which the instrument was caught in the severe up- or downdrafts (Fig. 8). When the fall rate deviated from the mean by more than 1.5 standard deviations, the predicted fall rate was used instead of the observed $dP/dt$.

The importance of this adjustment can be seen by considering AMP drop 15771 (Fig. A2). This drop was made through an updraft 60–80 m deep (Fig. 8). This updraft counterbalanced the tendency of the instrument to fall, and so the apparent fall rate is reduced and the initial estimate of the dissipation is very high (Fig. A2a, black line). However, assuming the instrument is mov-
ing through the water with the modeled flow speed, then a dissipation rate two orders of magnitude smaller results (gray line). The residual velocity between the observed fall rate and the modeled fall rate gives an updraft of 0.4 m s\(^{-1}\), which compares well to the vertical velocities observed by the C.S.S. Vector. The raw voltage output from the shear probes was examined. If the dissipation were really two orders of magnitude higher in this updraft, it would be apparent in the output. None was observed. Pitch and roll were also examined in this feature and were not found to be excessive. Fortunately, drop 15771 was the most extreme case, and most of the time the correction either was not applied or was very small. For instance, the upper water column of drop 15771 was also corrected, though much less, and the difference in the dissipation rate was typically only a factor of 2.

REFERENCES


